Improving isolated and in-context classification of handwritten characters

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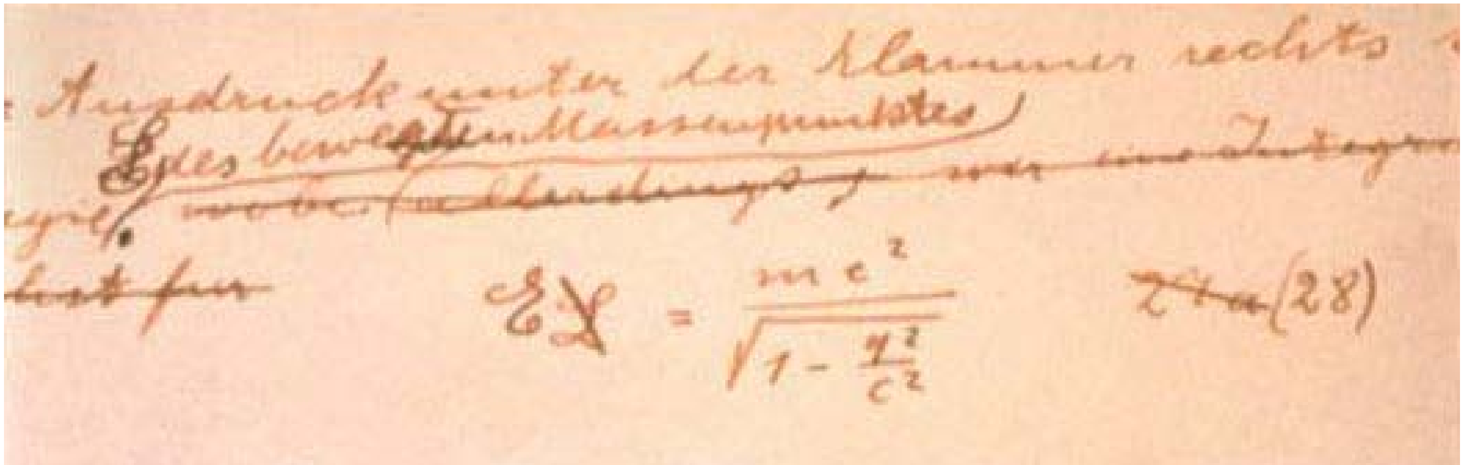
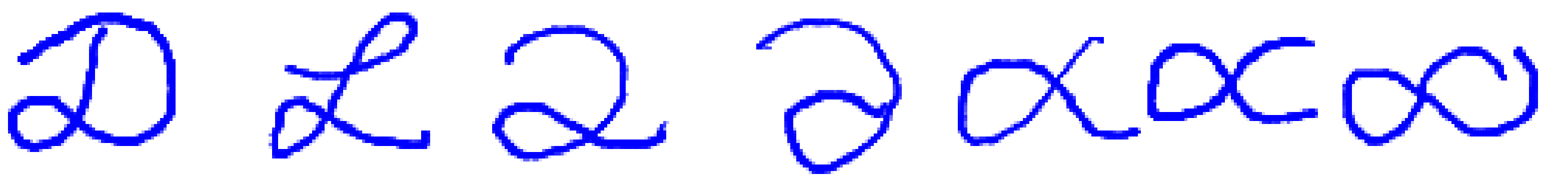
Pen-Based Math



- Input for CAS and document processing.
- 2D editing.
- Computer-mediated collaboration.

Pen-Based Math

- Different than natural language recognition:
  - 2-D layout is a combination of writing and drawing.
  - Many similar few-stroke characters.
  - Many alphabets, used idiosyncratically.
  - Many symbols, each person uses a subset.
  - No fixed dictionary for disambiguation.



Orthogonal Series Representation

- **Main idea**:
  Represent *c*oordinate curves as truncated orthogonal series.

- **Advantages**:
  - *Compact* – few coefficients needed
  - *Geometric*
    – the truncation order is a property of the character set
    – gives a natural metric on the space of characters
  - *Algebraic*
    – properties of curves can be computed algebraically
    (instead of numerically using heuristic parameters)
  - *Device independent*
    – resolution of the device is not important

Inner Product and Basis Functions

- Choose a functional inner product, e.g.

$$\langle f, g \rangle = \int_a^b f(t)g(t)w(t)dt$$

- This determines an orthonormal basis in the subspace of polynomials of degree *d.*

- Determine ϕ_i using GS on $\{1, t, t^2, t^3, \dots\}$.

- Can then approximate functions in subspaces

$$A(t) \approx \sum_{i=0}^{d} \alpha_i \phi_i(t) \qquad \alpha_i = \langle A(t), \phi_i(t) \rangle$$

First Look: Chebyshev Series

- Initially used Chebyshev series
  [Char+SMW ICDAR 2007].

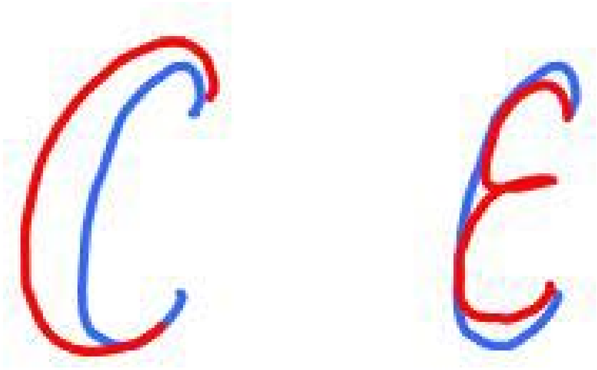
- Found could approximate closely
  (small RMS error) with series of order 10.

- Like symbols tended to form clusters.

Problems

- Want fast response –
  how to work while trace is being captured.

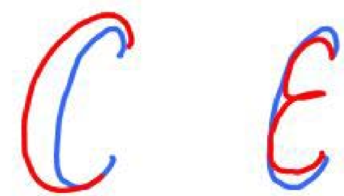
- Low RMS does not mean similar shape.



On-Line Series Coefficients

- Use Legendre polynomials P_i as basis on the interval $[-1,1]$, with weight function 1.
- Collect numerical values for $f(\lambda)$ on $[0, L]$.
  λ = arc length.
  L is not known until the pen is lifted.
- *As the sample points are collected*, numerically integrate the moments $\int \lambda^i f(\lambda) d\lambda$.
- After last point, compute series coefficients for f with domain and range scaled to $[-1,1]$.
  This uses a simple linear transformation of the moments.

Shape vs Variation



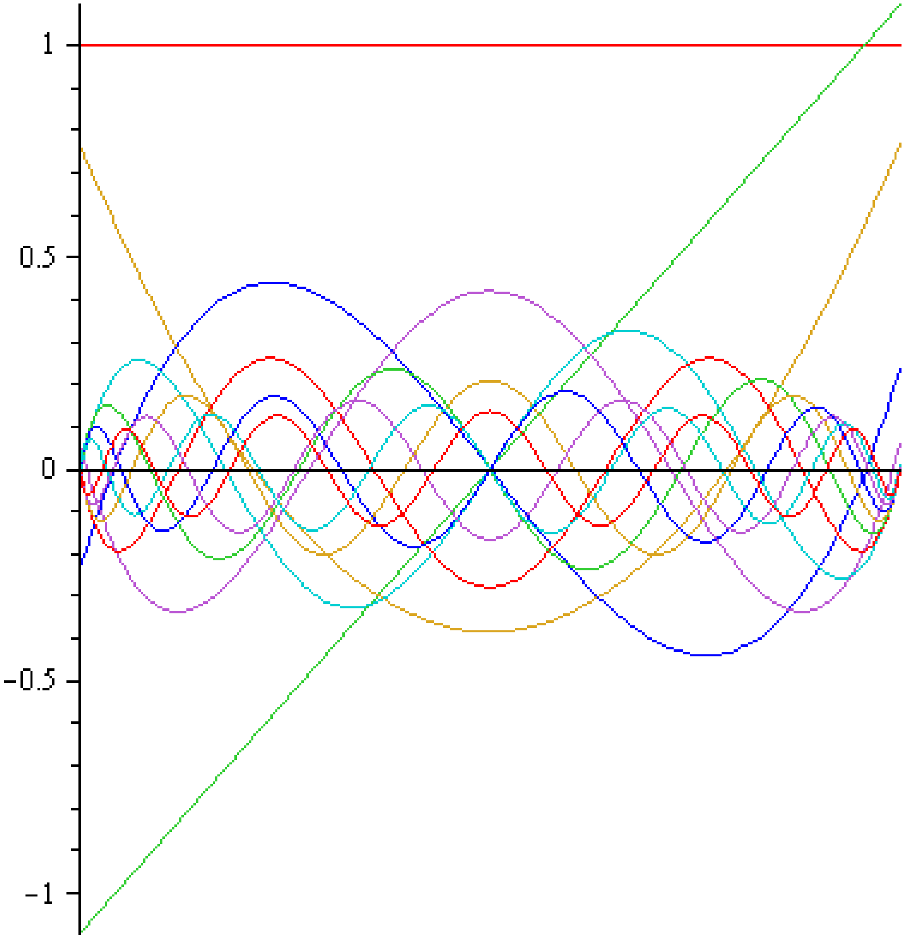
- The corners are not in the right places.
- Work in a jet space to force coords & derivatives close.
- Use a Legendre-Sobolev inner product

$$\langle f, g \rangle = \int_a^b f(t)g(t)dt + \mu_1 \int_a^b f'(t)g'(t)dt + \mu_2 \int_a^b f''(t)g''(t)dt + \cdots$$

- 1[st] jet space ⇒ set $\mu_i = 0$ for $i > 1$.
  - Choose μ_1 experimentally to maximize reco rate.
  - Can be also done on-line.

    [Golubitsky + SMW 2008, 2009]

Legendre-Sobolev Basis



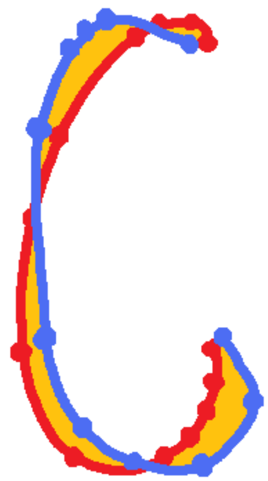
$$a = 0, b = 1, \mu = .125$$

Life in an Inner Product Space

- With the Legendre-Sobolev inner product we have
  - Low dimensional rep for curves (10 + 10 + 1)
  - Compact rep of samples ~ 160 bits [G+W 2009]
  - **>99% linear separability => convexity of classes**
  - A useful notion of distance between curves
    that is very fast to compute

Distance Between Curves

- **Elastic matching:**
- Approximate the variation between curves by some fn of distances between sample points.
- May be coordinate curves or curves in a jet space.



- Sequence alignment
- Interpolation ("resampling")

- **Why not just calculate the area?**
- This is very fast in ortho. series representation.

Distance Between Curves

$$\bar{x}(t) = x(t) + \xi(t) \qquad \xi(t) = \sum_{i=0}^{\infty} \xi_i \phi_i(t), \qquad \phi_i \text{ ortho on } [a,b] \text{ with } w(t) = 1.$$

$$\bar{y}(t) = y(t) + \eta(t) \qquad \eta(t) = \sum_{i=0}^{\infty} \eta_i \phi_i(t)$$

$$\rho^2(C, \bar{C}) = \int_a^b \left[\left(x(t) - \bar{x}(t) \right)^2 + \left(y(t) - \bar{y}(t) \right)^2 \right] dt$$

$$= \int_a^b [\xi(t)^2 + \eta(t)^2] dt$$

$$\approx \int_a^b \left[\sum_{i=0}^{d} \xi_i^2 \phi_i^2(t) + \text{cross terms} + \sum_{i=0}^{d} \eta_i^2 \phi_i^2(t) + \text{cross terms} \right] dt$$

$$= \sum_{i=0}^{d} \xi_i^2 + \sum_{i=0}^{d} \eta_i^2$$

Comparison of Candidate to Models

- Use Euclidean distance in the coefficient space.

- *Just as accurate* as elastic matching.
- *Much less expensive.*
- Linear in d, the degree of the approximation.
  < 3 d machine instructions (30ns) *vs* several thousand!
- Can trace through SVM-induced cells incrementally.

- Normed space for characters gives other advantages.

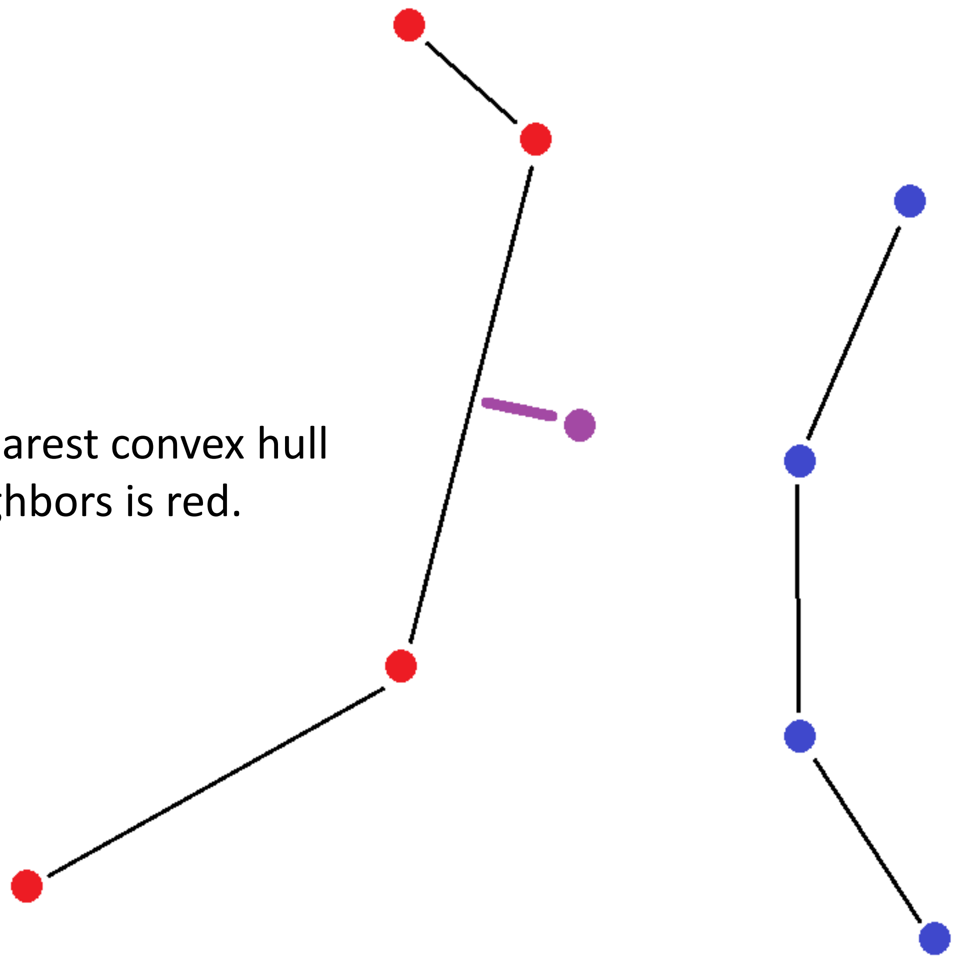
The Joy of Convex

- Linear separability ⇒ Convexity
  ⇒ Linear homotopies stay within a class

$$C = (1- t)\,A + t\,B$$

- Can compute distance of a sample to this line
- Distance to convex hull of nearest neighbours in class gives best recognition [Golubitsky+SMW 2009,2010]

Choosing between Alternatives



The nearest convex hull of neighbors is red.

Transformation-independent recognition

- Rotation- and shear-independent recognition can be achieved by computing invariant functions from the coordinates of a character.

- Then, similar to the coordinate functions, the invariants can be approximated with orthogonal polynomials and their coefficients used in recognition.

- We have shown in the past, that integral invariants can be a suitable descriptor of online handwritten characters.

The main contributions of this work

- Optimization of isolated character recognition by adjusting the
  - "jet scale" in the LS inner product for coordinate functions and integral invariants
  - degree of approximation of coordinate functions and integral invariants.
- An in-context rotation-invariant algorithm that yields substantially better results than isolated recognition and can be extended to other transformations.

Coordinate Functions

- The optimal μ for approximation of coordinate functions was found to minimize classification error.
- The original characters in the dataset were considered without any distortions.
- The characters were approximated with corresponding values of μ in the range from 0 to 0.2.

Complexity of Handwritten Characters

- We considered the possibility that the optimal value of μ might depend on the nature of the characters to be recognized.
- We took the notion of a sample's complexity as

$$\eta = \sum_{i=1}^{d} (X_i^{1/i} + Y_i^{1/i}), |X_i| \leq 1 \text{ and } |Y_i| \leq 1$$

  where X_i and Y_i are normalized coefficients of approximation of the sample with orthogonal polynomials
- The sample complexity function is derived from the fact that coefficients of higher degree are typically greater for "complex" characters – characters that contain large number of loops and/or amount of curvature.

In-context transformation-invariant recognition

- Recognition of distorted math symbols without context is sometimes impossible

$$\text{“}<\text{” vs } \angle, \text{ “}|\text{” vs “}/\text{”, “}\cup\text{” vs “}\subset$$

- It has been shown that n-grams provide useful semantic information in a mathematical setting.
- We use context to improve distortion-independent classification, taking advantage of the fact that samples written by a person typically exhibit similar degree of transformation.
- We consider the case of rotation (shear and other transformations may be handled similarly).

Experimental setting

- The testing set was represented in InkML format.
- It contains 50,703 characters in 242 classes.
- Class labels incorporated the number of strokes (therefore, single-stroke and double-stroke “7” were considered as different classes).

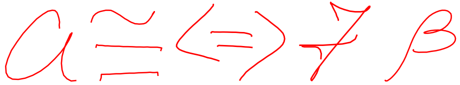


Figure: Characters from the training dataset

- The model was trained with non-transformed samples.
- For the recognition phase, sequences of n characters were taken from the dataset and each sequence was rotated by a random angle $\gamma \in [-\beta, \beta]$.

Coordinate functions

- The figure shows the error of recognition using coefficients of X and Y.

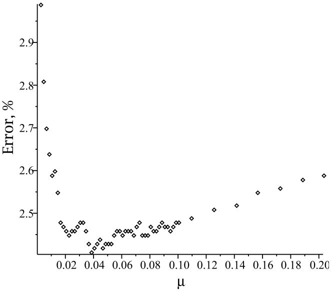
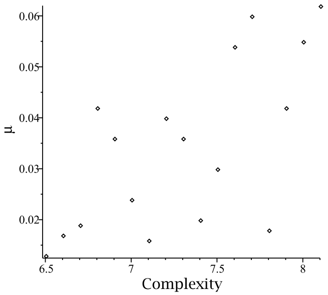


Figure: Recognition error of non-transformed characters for different values of μ

- $\approx 2.4\%$ error is reached for $\mu = 0.04$.

Optimal μ for characters with different complexities

- We found that the optimal μ is not strongly correlated with the complexity of characters.



- Results of Spearman and Kendall tau-a correlation tests between complexity and μ are respectively: $\rho_{\mu,\eta}(13) = 0.52, p = 0.047$ and $\tau_{\mu,\eta}(13) = 0.38, p = 0.053$.

I_0

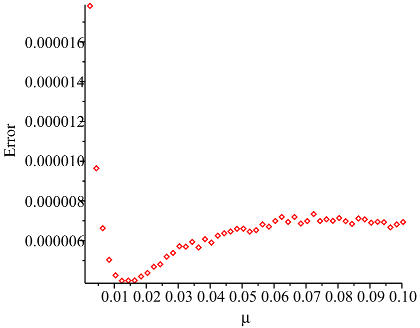
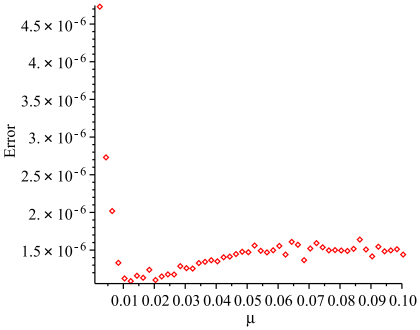


Figure: Average maximum error in coefficients of I_0 depending on μ

I_1



(a)

Figure: Average maximum error in coefficients of I_1 depending on μ

Degree of Approximation

Table: Recognition error (Rec.Err.), maximum approximation error (Max.Err.) and average relative approximation error of coordinates (Avg.Err.) for different degrees of approximation d, $\mu = 0.04$

d	9	10	11	12	13	14	15
Rec.Err. %	2.57	2.49	2.46	2.43	2.44	2.45	2.46
Max.Err.	707	539	539	484	475	494	500
Avg.Err.($\times 10^{-3}$)	1.9	1.6	1.4	1.2	1.1	1.0	1.2

We find degree 12 to be the optimum for recognition of symbols in our collection.

In-Context Classification: Parameters

There are 3 parameters that in-context recognition rate can depend on:

- Number p of closest classes in computation of error likelihood.
- Rotation angle.
- Size n of the set of characters.

Evaluatoin of p

- We fixed the parameter $n = 3$.
- Performed classification for values p of 2, 3 and 4.
- We found that p has almost no effect on recognition error.
- We took $p = 3$ and continued the experiments.

Evaluation of n and the rotation angle

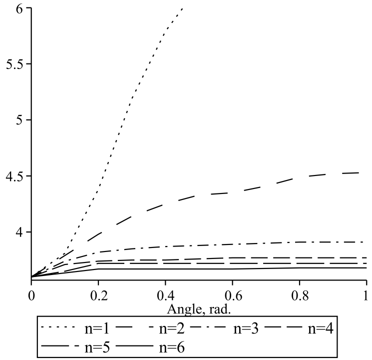


Figure: Recognition error (%) for different size of context n and different angles of rotation (in radians)

A significant reduction in error is achieved between $n = 1$ and $n = 3$.

Contributions of the presented work

- The optimal range of values for the jet scale for coordinate basis functions.
- A study that demonstrates that this optimal value of μ, to a first approximation, does not depend on the complexity of the characters tested.
- The optimal values for the jet scale for the integral invariants I_0 and I_1, used in rotation-independent recognition.
- The optimal degree of the approximating series.
- An algorithm for orientation-invariant in-context classification by considering characters in groups.